Analysis and Design of Algorithms
Lecture 5

Sorting Algorithms II

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Counting Sort

Counting Sort

- **Counting sort** is a sorting technique based on keys between a specific range. It works by counting the number of objects having distinct key values (kind of hashing). Then doing some arithmetic to calculate the position of each object in the output sequence.

Counting Sort

❑ Algorithm:

- **Step1**: Create a count array to store the count of each unique object
- **Step2** : Modify count array by adding the previous number.
- **Step3** : Create output array by decrease count array

Counting Sort

- Example 1 Assume the following Array in range of 0 to 5:

1	**4**	**3**	**2**	**3**	**5**	**2**

Counting Sort

- Create a count array to store the count of each unique object:

1	**4**	**3**	**2**	**3**	**5**	**2**

Counting Sort

- Create a count array to store the count of each unique object:

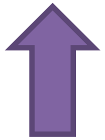
1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	0	0	0	0

Counting Sort

- ❑ Create a count array to store the count of each unique object:

1	4	3	2	3	5	2



0	1	2	3	4	5
0	0	0	0	0	0

Counting Sort

- ❑ Create a count array to store the count of each unique object:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	0	0	0	0

Counting Sort

- Create a count array to store the count of each unique object:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	0	0	0	0

Counting Sort

❑ Create a count array to store the count of each unique object:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	0	0	0	0

Counting Sort

- Create a count array to store the count of each unique object:

1	**4**	**3**	**2**	**3**	**5**	**2**

0	**1**	**2**	**3**	**4**	**5**
0	1	0	0	0	0

Counting Sort

- ❑ Create a count array to store the count of each unique object:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	0	0	0	0

Counting Sort

❑ Create a count array to store the count of each unique object:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	0	0	1	0

Counting Sort

- ❑ Create a count array to store the count of each unique object:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	0	0	1	0

Counting Sort

❑ Create a count array to store the count of each unique object:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	0	0	1	0

Counting Sort

- ❑ Create a count array to store the count of each unique object:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	0	1	1	0

Counting Sort

- Create a count array to store the count of each unique object:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	0	1	1	0

Counting Sort

- Create a count array to store the count of each unique object:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	0	1	1	0

Counting Sort

- ❑ Create a count array to store the count of each unique object:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	1	1	1	0

Counting Sort

- ❑ Create a count array to store the count of each unique object:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	1	1	1	0

Counting Sort

- Create a count array to store the count of each unique object:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	1	2	1	0

Counting Sort

- ❑ Create a count array to store the count of each unique object:

1	**4**	**3**	**2**	**3**	**5**	**2**

0	**1**	**2**	**3**	**4**	**5**
0	1	1	2	1	0

Counting Sort

- ❑ Create a count array to store the count of each unique object:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	1	2	1	0

Counting Sort

- ❑ Create a count array to store the count of each unique object:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	1	2	1	1

Counting Sort

- Create a count array to store the count of each unique object:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	1	2	1	1

Counting Sort

❑ Create a count array to store the count of each unique object:

1	**4**	**3**	**2**	**3**	**5**	**2**

0	**1**	**2**	**3**	**4**	**5**
0	1	1	2	1	1

Counting Sort

- Create a count array to store the count of each unique object:

1	**4**	**3**	**2**	**3**	**5**	**2**

0	**1**	**2**	**3**	**4**	**5**
0	1	2	2	1	1

Counting Sort

- ❑ Modify count array by adding the previous number :

1	**4**	**3**	**2**	**3**	**5**	**2**

0	**1**	**2**	**3**	**4**	**5**
0	1	2	2	1	1

Counting Sort

- Modify count array by adding the previous number :

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	2	2	1	1

Counting Sort

- Modify count array by adding the previous number :

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	3	2	1	1

Counting Sort

- Modify count array by adding the previous number :

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	3	5	1	1

Counting Sort

- Modify count array by adding the previous number :

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	3	5	6	1

Counting Sort

- Modify count array by adding the previous number :

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	3	5	6	7

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	3	5	6	7

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	3	5	6	7

1	2	3	4	5	6	7

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	3	5	6	7

1	2	3	4	5	6	7

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	3	5	6	7

1	2	3	4	5	6	7

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	1	3	5	6	7

1	2	3	4	5	6	7
1						

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	3	5	6	7

1	2	3	4	5	6	7
1						

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	3	5	6	7

1	2	3	4	5	6	7
1						

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	3	5	5	7

1	2	3	4	5	6	7
1					4	

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	3	5	5	7

1	2	3	4	5	6	7
1					4	

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	3	5	5	7

1	2	3	4	5	6	7
1					4	

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	3	5	5	7

1	2	3	4	5	6	7
1				3	4	

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	3	4	5	7

1	2	3	4	5	6	7
1				3	4	

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	3	4	5	7

1	2	3	4	5	6	7
1				3	4	

Counting Sort

- ❑ Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	3	4	5	7

1	2	3	4	5	6	7
1				3	4	

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	3	4	5	7

1	2	3	4	5	6	7
1		2		3	4	

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	2	4	5	7

1	2	3	4	5	6	7
1		2		3	4	

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	2	4	5	7

1	2	3	4	5	6	7
1		2		3	4	

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	2	4	5	7

1	2	3	4	5	6	7
1		2		3	4	

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	2	4	5	7

1	2	3	4	5	6	7
1		2	3	3	4	

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	2	3	5	7

1	2	3	4	5	6	7
1		2	3	3	4	

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	2	3	5	7

1	2	3	4	5	6	7
1		2	3	3	4	

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	2	3	5	7

1	2	3	4	5	6	7
1		2	3	3	4	

Counting Sort

- ❑ Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	2	3	5	7

1	2	3	4	5	6	7
1		2	3	3	4	5

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	2	3	5	6

1	2	3	4	5	6	7
1		2	3	3	4	5

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	2	3	5	6

1	2	3	4	5	6	7
1		2	3	3	4	5

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	2	3	5	6

1	2	3	4	5	6	7
1		2	3	3	4	5

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	2	3	5	6

1	2	3	4	5	6	7
1	2	2	3	3	4	5

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	1	3	5	6

1	2	3	4	5	6	7
1	2	2	3	3	4	5

Counting Sort

- Output each object from the input sequence followed by decreasing its count by 1:

1	4	3	2	3	5	2

0	1	2	3	4	5
0	0	1	3	5	6

1	2	3	4	5	6	7
1	2	2	3	3	4	5

Counting Sort

- Array is now sorted

1	2	2	3	3	4	5

Counting Sort

❑ Example 2:

Range=[0-4]

2	4	1	1	3

Count

0	1	2	3	4
0	2	1	1	1

Add

0	1	2	3	4
0	2	3	4	5

Output

1	2	3	4	5
		2		

Reduce

0	1	2	3	4
0	2	2	4	5

Output

1	2	3	4	5
		2		4

Reduce

0	1	2	3	4
0	2	2	4	4

Output

1	2	3	4	5
	1	2		4

Reduce

0	1	2	3	4
0	1	2	4	4

Output

1	2	3	4	5
1	1	2		4

Reduce

0	1	2	3	4
0	0	2	4	4

Output

1	2	3	4	5
1	1	2	3	4

Sorted

1	2	3	4	5
1	1	2	3	4

Counting Sort

❑ Python

Code

```
def countSort(arr):
    outputarr = [0 for i in range(127)]
    countarr = [0 for i in range(127)]

    for i in arr:
        countarr[ord(i)] += 1

    for i in range(127):
        countarr[i] += countarr[i-1]

    for i in range(len(arr)):
        outputarr[countarr[ord(arr[i])]-1] = arr[i]
        countarr[ord(arr[i])] -= 1

    return outputarr[0:len(arr)]
```

ASCII Table

Dec	Hex	Oct	Char	Dec	Hex	Oct	Char	Dec	Hex	Oct	Char	Dec	Hex	Oct	Char
0	0	0		32	20	40	[space]	64	40	100	@	96	60	140	`
1	1	1		33	21	41	!	65	41	101	A	97	61	141	a
2	2	2		34	22	42	"	66	42	102	B	98	62	142	b
3	3	3		35	23	43	#	67	43	103	C	99	63	143	c
4	4	4		36	24	44	$	68	44	104	D	100	64	144	d
5	5	5		37	25	45	%	69	45	105	E	101	65	145	e
6	6	6		38	26	46	&	70	46	106	F	102	66	146	f
7	7	7		39	27	47	'	71	47	107	G	103	67	147	g
8	8	10		40	28	50	(	72	48	110	H	104	68	150	h
9	9	11		41	29	51	)	73	49	111	I	105	69	151	i
10	A	12		42	2A	52	*	74	4A	112	J	106	6A	152	j
11	B	13		43	2B	53	+	75	4B	113	K	107	6B	153	k
12	C	14		44	2C	54	,	76	4C	114	L	108	6C	154	l
13	D	15		45	2D	55	-	77	4D	115	M	109	6D	155	m
14	E	16		46	2E	56	.	78	4E	116	N	110	6E	156	n
15	F	17		47	2F	57	/	79	4F	117	O	111	6F	157	o
16	10	20		48	30	60	0	80	50	120	P	112	70	160	p
17	11	21		49	31	61	1	81	51	121	Q	113	71	161	q
18	12	22		50	32	62	2	82	52	122	R	114	72	162	r
19	13	23		51	33	63	3	83	53	123	S	115	73	163	s
20	14	24		52	34	64	4	84	54	124	T	116	74	164	t
21	15	25		53	35	65	5	85	55	125	U	117	75	165	u
22	16	26		54	36	66	6	86	56	126	V	118	76	166	v
23	17	27		55	37	67	7	87	57	127	W	119	77	167	w
24	18	30		56	38	70	8	88	58	130	X	120	78	170	x
25	19	31		57	39	71	9	89	59	131	Y	121	79	171	y
26	1A	32		58	3A	72	:	90	5A	132	Z	122	7A	172	z
27	1B	33		59	3B	73	;	91	5B	133	[	123	7B	173	{
28	1C	34		60	3C	74	<	92	5C	134	\	124	7C	174	|
29	1D	35		61	3D	75	=	93	5D	135	]	125	7D	175	}
30	1E	36		62	3E	76	>	94	5E	136	^	126	7E	176	~
31	1F	37		63	3F	77	?	95	5F	137	_	127	7F	177	

Counting Sort

```
arr = "mynameiskhan"
ans = countSort(arr)
print("".join(ans))
```

Counting Sort

- **Time Complexity:** O(n+k) where n is the number of elements in input array, and k is the range of input.

- Example of worst case
  - Range between 1 to 10K

10	**5**	**10k**	**5k**	**200**

Radix Sort

Radix Sort

- Radix sort is an algorithm that sorts numbers by processing digits of each number either starting from the least significant digit (LSD) or starting from the most significant digit (MSD).
- The idea of Radix Sort is to do digit by digit sort starting from least significant digit to most significant digit. Radix sort uses counting sort as a subroutine to sort.

Radix Sort

❑ Algorithm:

- **Step1**: Take the least significant digit of each element
- **Step2** : Sort the list of elements based on that digit
- **Step3** : Repeat the sort with each more significant digit

Radix Sort

- Assume the following Array:

170	**45**	**75**	**90**	**802**	**24**	**2**	**66**

- The Sorted list will appear after three steps

170	45	75	90	802	24	2	66
170	90	802	2	24	45	75	66
802	2	24	45	66	170	75	90
2	24	45	66	75	90	170	802

Radix Sort

- Step1: Sorting by least significant digit (1s place)

170	45	75	90	802	24	2	66

170	90	802	2	24	45	75	66

Radix Sort

- Step2: Sorting by next digit (10s place)

170	90	802	2	24	45	75	66

802	2	24	45	66	170	75	90

Radix Sort

- Step3: Sorting by most significant digit (100s place)

802	2	24	45	66	170	75	90

2	24	45	66	75	90	170	802

Radix Sort

- Array is now sorted

2	24	45	66	75	90	170	802

Radix Sort

- Example 2

1 2 3		1 2 3		1 2 3		1 2 3
5 8 3		5 8 3		6 2 5		1 5 4
1 5 4		1 5 4		1 5 4		4 5 6
5 6 7	1s →	6 2 5	10s →	4 5 6	100s →	5 6 7
6 8 9		4 5 6		5 6 7		5 8 3
6 2 5		5 6 7		5 8 3		6 2 5
4 5 6		6 8 9		6 8 9		6 8 9

Radix Sort

```
def countingSort(arr, count1):
    n = len(arr)
    output = [0] * (n)
    count = [0] * (10)
    for i in range(0, n):
        index = (arr[i]/count1)
        count[ int((index)%10) ] += 1

    for i in range(1,10):
        count[i] += count[i-1]

    i = n-1
    while i>=0:
        index = (arr[i]/count1)
        output[ count[ int((index)%10) ] - 1] = arr[i]
        count[ int((index)%10) ] -= 1
        i -= 1
    return output
```

❑ Python Code

```
def radixSort(arr):
    # Find the maximum number to know number of digits
    maxnum = max(arr)

    count = 1
    while maxnum/count > 0:
        arr=countingSort(arr,count)
        count *= 10
    return arr
```

Radix Sort

❑ Python Code

```
arr = [ 170, 45, 75, 90, 802, 24, 2, 66]
print(radixSort(arr))
```

- **Time Complexity:** $O(n+k/d)$ where n is the number of elements in input array, k is the range of input, and d is number of digits.

Merge Sort

Merge Sort

- **Merge Sort** is a **Divide and Conquer algorithm**. It divides input array in two halves, calls itself for the two halves and then merges the two sorted halves.

❑ Algorithm:

- **Step1**: Divide the list recursively into two halves until it can no more be divided
- **Step2** : Merge (**Conquer**) the smaller lists into new list in sorted order

Merge Sort

- Assume the following Array:

85	**24**	**63**	**45**	**17**	**31**	**96**	**50**

Merge Sort

- Divide

85	24	63	45	17	31	96	50

85	24	63	45

17	31	96	50

Merge Sort

- Divide

85	24	63	45	17	31	96	50

85	24	63	45

17	31	96	50

85	24

63	45

17	31

96	50

Merge Sort

- Divide

85	24	63	45	17	31	96	50

85	24	63	45

17	31	96	50

85	24

63	45

17	31

96	50

85

24

63

45

17

31

96

50

Merge Sort

- ❑ Sort & Merge

85	**24**	**63**	**45**	**17**	**31**	**96**	**50**

Merge Sort

- Sort & Merge

85	24	63	45	17	31	96	50

Merge Sort

- Sort & Merge

24	85

85	24	63	45	17	31	96	50

Merge Sort

- Sort & Merge

24	85		

85	24	63	45	17	31	96	50

Merge Sort

- Sort & Merge

24	85	45	63

85	24	63	45	17	31	96	50

Merge Sort

- ❑ Sort & Merge

24	85	45	63		

85	24	63	45	17	31	96	50

Merge Sort

- Sort & Merge

24	85	45	63	17	31

85	24	63	45	17	31	96	50

Merge Sort

- Sort & Merge

24	85	45	63	17	31		
85	24	63	45	17	31	96	50

Merge Sort

- Sort & Merge

24	85	45	63	17	31	50	96
85	24	63	45	17	31	96	50

Merge Sort

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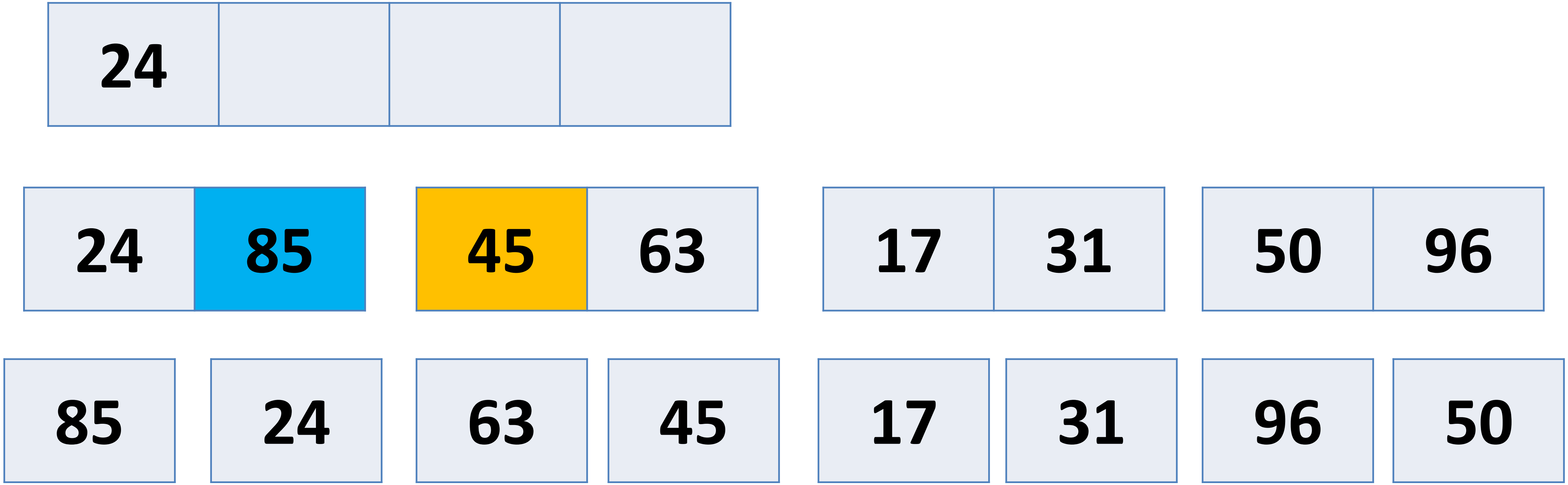
24	85	45	63	17	31	50	96

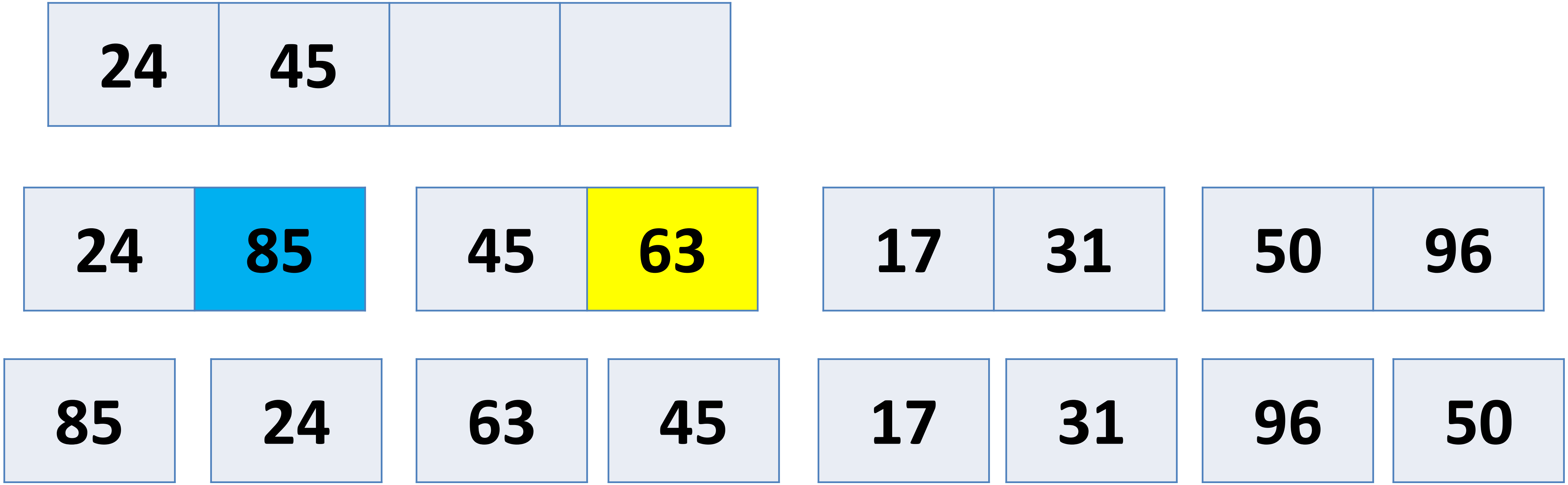
85	24	63	45	17	31	96	50

Merge Sort

- ❑ Sort & Merge



- Sort & Merge



Merge Sort

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24	45	63	

24	85	45	63	17	31	50	96

85	24	63	45	17	31	96	50

Merge Sort

- ❑ Sort & Merge

24	45	63	85

24	85	45	63	17	31	50	96

85	24	63	45	17	31	96	50

Merge Sort

- ❑ Sort & Merge

24	45	63	85	17	31	50	96

24	85	45	63	17	31	50	96

85	24	63	45	17	31	96	50

- Sort & Merge

17	24	31	45	50	63	85	96

24	45	63	85

17	31	50	96

24	85

45	63

17	31

50	96

85 | 24 | 63 | 45 | 17 | 31 | 96 | 50

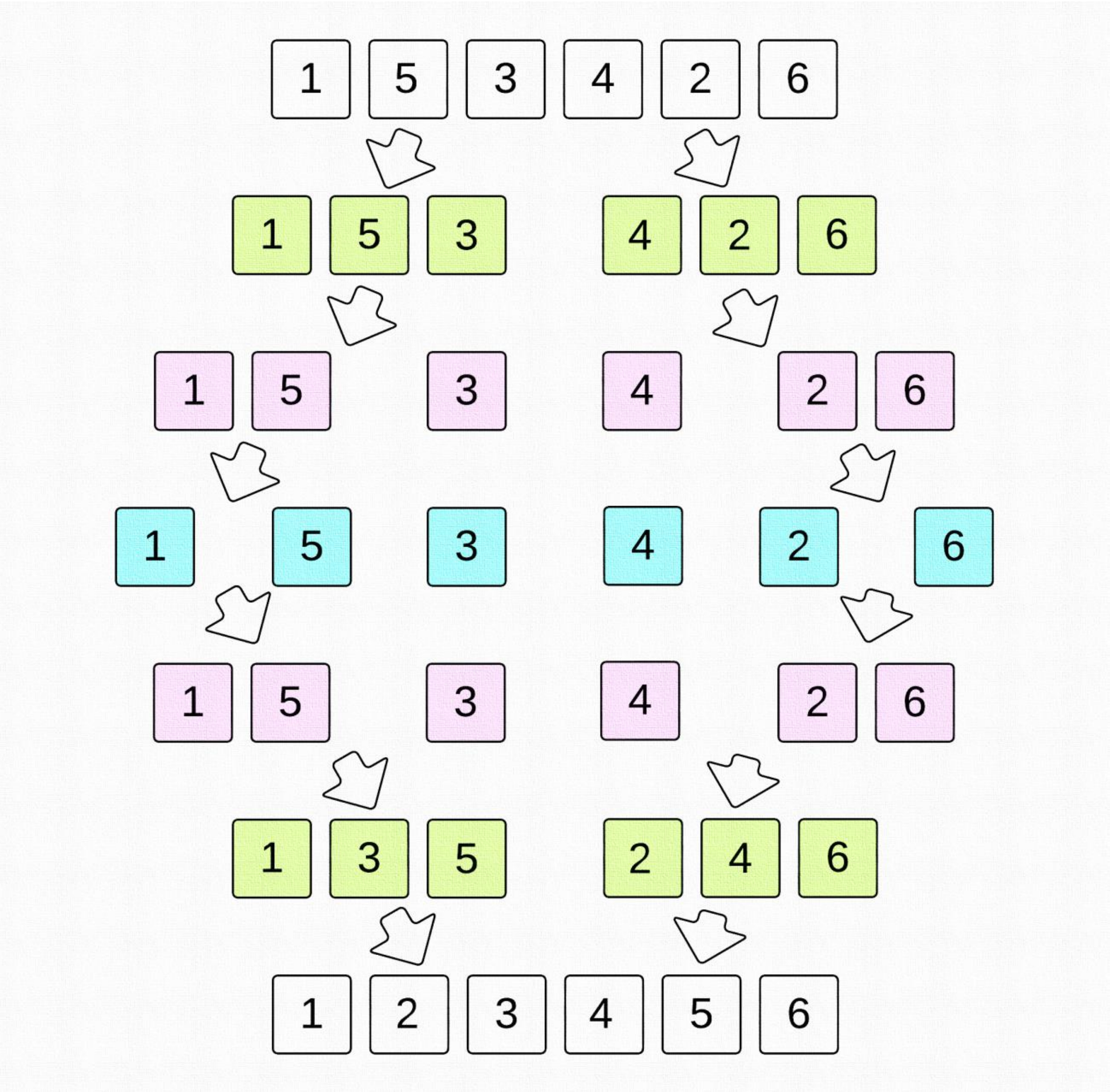
Merge Sort

- ❑ Array is now sorted

17	24	31	45	50	63	85	96

Merge Sort

- ❑ Example 2



Merge Sort

```
def merge(arr, l, m, r):
    n1 = m - l + 1
    n2 = r- m
    L = [0] * (n1)
    R = [0] * (n2)
    for i in range(0 , n1):
        L[i] = arr[l + i]
    for j in range(0 , n2):
        R[j] = arr[m + 1 + j]
    i = 0     # Initial index of first subarray
    j = 0     # Initial index of second subarray
    k = l     # Initial index of merged subarray
    while i < n1 and j < n2 :
        if L[i] <= R[j]:
            arr[k] = L[i]
            i += 1
        else:
            arr[k] = R[j]
            j += 1
        k += 1
    while i < n1: # Copy the remaining elements of L[]
        arr[k] = L[i]
        i += 1
        k += 1
    while j < n2: # Copy the remaining elements of R[]
        arr[k] = R[j]
        j += 1
        k += 1
```

Merge Sort

```
def mergeSort(arr,l,r):
    if l < r:
        m = int((l+(r-1))/2)
        mergeSort(arr, l, m)
        mergeSort(arr, m+1, r)
        merge(arr, l, m, r)
    return arr
```

```
arr = [12, 11, 13, 5, 6, 7]
print(mergeSort(arr,0,len(arr)-1))
```

Merge Sort

- **Time Complexity:** $O(n * \log(n))$

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THANKS FOR YOUR TIME